wortkoelen1a.xmcd

$\lambda := 1.4$ $\Delta t := 80$ $mm := 10^{-3}$ $kw := 1000$

$d := 1mm$

$A\_per\_plaat := 0.01$ $platen := 36$

$segmenten := 10$ aantal segmenten voor de berekening opsplitsen

$$P = \lambda \cdot A \cdot \frac{\Delta Temp}{d}$$

$$psegment := \lambda \cdot \frac{A\_per\_plaat \cdot platen}{segmenten} \cdot \frac{\Delta t}{d}$$

$$psegment = 4.032 \cdot kw$$

$$P = -sw \cdot m \cdot \left(\frac{d}{dt} Temp\right)$$

$$\frac{d}{dt} Temp = \frac{P}{sw \cdot m}$$

$$\frac{d}{dt} Temp = \frac{-\lambda \cdot A \cdot \frac{\Delta Temp}{d}}{sw \cdot m}$$

$$\frac{d}{dt} Tempwater = \frac{A \cdot \lambda \cdot (Tempwort - tempwater)}{d \cdot m \cdot sw}$$

$$\frac{d}{dt} Tempwort = \frac{A \cdot \lambda \cdot (Tempwater - tempwort)}{d \cdot m \cdot sw}$$

http://www.maths.surrey.ac.uk/explore/vithyaspages/coupled.html

$$\frac{d}{dt} Tempwort = \frac{A \cdot \lambda \cdot (Tempwater - tempwort)}{d \cdot mwort \cdot sw} \qquad b = \frac{A \cdot \lambda}{d \cdot mwort \cdot sw}$$

$$\frac{d}{dt} Tempwater = \frac{A \cdot \lambda \cdot (Tempwort - tempwater)}{d \cdot mwater \cdot sw} \qquad a = \frac{A \cdot \lambda}{d \cdot mwater \cdot sw}$$

$$\frac{d}{dt}y = b\cdot x - b\cdot y \qquad [1]$$

$$\frac{d}{dt}x = -a\cdot x + a\cdot y \qquad [2]$$

$x = \text{Tempwater}$

$y = \text{Tempwort}$

Step 1: First make x the subject of (1) in order to eliminate all x related items

$$x = \frac{\frac{d}{dt}y + b\cdot y}{b}$$

Step 2: Substitute in (2) to get

$$\frac{d}{dt}\frac{\frac{d}{dt}y + b\cdot y}{b} = -a\cdot\frac{\frac{d}{dt}y + b\cdot y}{b} + a\cdot y$$ levert

$$\frac{1}{b}\cdot\frac{d^2}{dt^2}y + \frac{d}{dt}y + \frac{a}{b}\cdot\frac{d}{dt}y + a\cdot y - a\cdot y = 0$$ levert

$$\frac{1}{b}\cdot\frac{d^2}{dt^2}y + \frac{d}{dt}y\cdot\left(1 + \frac{a}{b}\right) = 0$$

Step 3: The roots of the auxiliary equation

$$\frac{m^2}{b} + \left(\frac{a}{b} + 1\right)\cdot m = 0$$ are

$$\begin{pmatrix} 0 \\ -a - b \end{pmatrix}$$

The homogeneous problem is

$$y = A1 \cdot e^{0} + A2 \cdot e^{-(a+b) \cdot t}$$

beginvoorwaarden

$y0$ levert

$$y0 = A1 + A2$$

using

$$x = \frac{\frac{d}{dt}y + b \cdot y}{b}$$ gives

$$x = -A2 \cdot \frac{(a+b)}{b} \cdot e^{-(a+b) \cdot t} + \left[A1 + A2 \cdot e^{-(a+b) \cdot t}\right]$$

$$x = A1 + e^{-(a+b) \cdot t} \cdot \left[A2 \cdot \left[1 - \frac{(a+b)}{b}\right]\right]$$

x0 bwginvoorwaarde levert

$$x0 = A1 + \left[A2 \cdot \left[1 - \frac{(a+b)}{b}\right]\right]$$

$$y0 = A1 + A2$$

$$A1 = y0 - A2$$

$$x0 = y0 - A2 + \left[A2 \cdot \left[1 - \frac{(a+b)}{b}\right]\right]$$ geeft

$$-\frac{b \cdot (x0 - y0)}{a + b}$$

$$A2 = \frac{b \cdot (y0 - x0)}{a + b}$$

$$A1 = y0 - \frac{b \cdot (y0 - x0)}{a + b}$$

$$x = y0 - \frac{b\cdot(y0 - x0)}{a + b} + e^{-(a+b)\cdot t}\cdot\left[\frac{b\cdot(y0 - x0)}{a + b}\cdot\left[1 - \frac{(a + b)}{b}\right]\right]$$

$$y = \left[y0 - \frac{b\cdot(y0 - x0)}{a + b}\right]\cdot e^{0} + \frac{b\cdot(y0 - x0)}{a + b}\cdot e^{-(a+b)\cdot t}$$

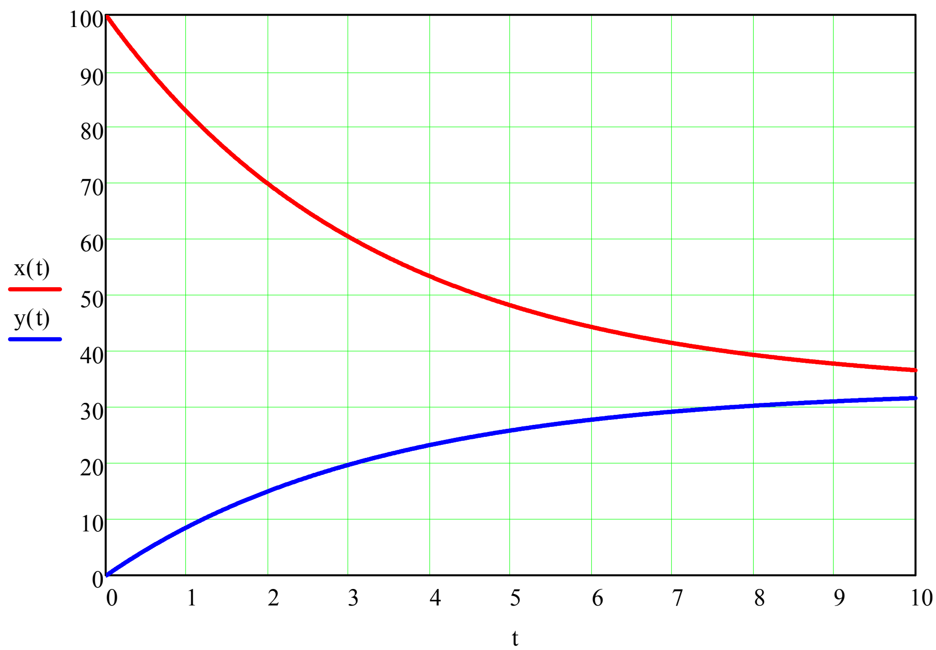
$a := 0.2$ $\quad b := 0.1$ $\quad x0 := 100$ $\quad y0 := 0$

$$x(t) := y0 - \frac{b\cdot(y0 - x0)}{a + b} + e^{-(a+b)\cdot t}\cdot\left[\frac{b\cdot(y0 - x0)}{a + b}\cdot\left[1 - \frac{(a + b)}{b}\right]\right]$$

$$y(t) := \left[y0 - \frac{b\cdot(y0 - x0)}{a + b}\right]\cdot e^{0} + \frac{b\cdot(y0 - x0)}{a + b}\cdot e^{-(a+b)\cdot t}$$

$y(5) = 25.896$

$t := 0, 0.01 .. 10$



λ