

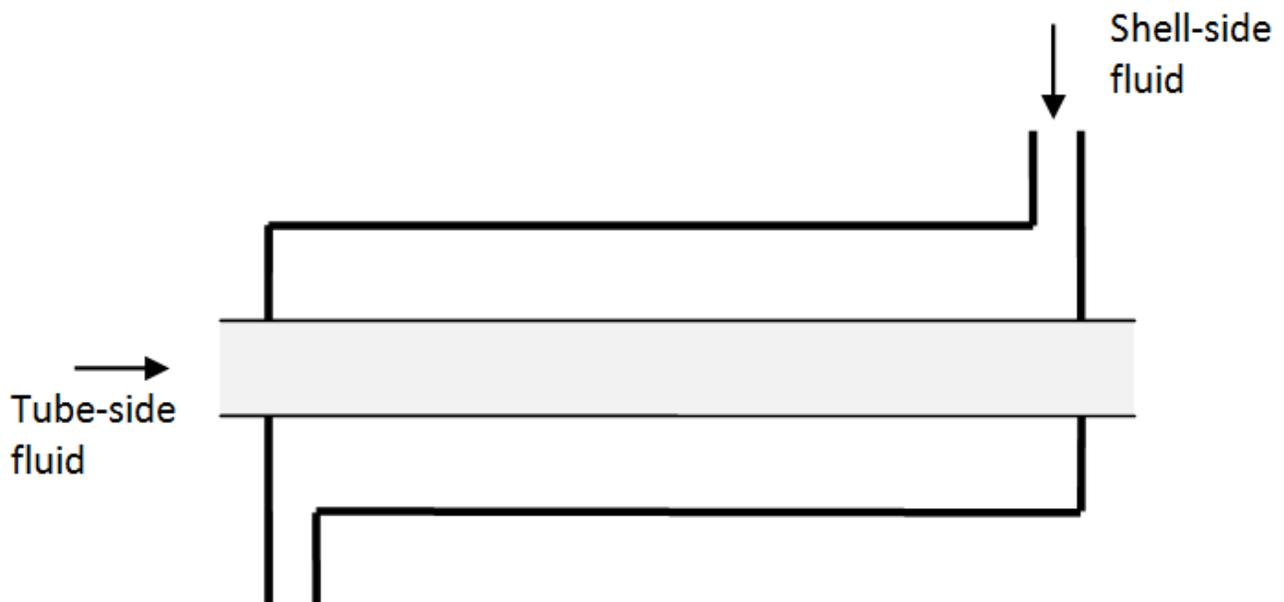
# Countercurrent Double-Pipe Heat Exchanger

## Introduction

This application models the temperature dynamics of a countercurrent double pipe heat exchanger. Three partial differential equations describe

- heat balances across the tube- and shell-side liquids,
- and a heat balance across the tube-wall (taking into account the heat flow from the shell- and tube-side liquids, and conduction along the length of the tube)

The equations are solved numerically, and the temperature profiles are plotted. The heat exchanger is assumed to be perfectly insulated. Densities, specific heat capacities, heat transfer coefficients, and thermal conductivities are assumed to be constant.



## Parameters

Specific heat capacities for the tube-side fluid, shell-side fluid and the tube wall

>  $C_{pt} := 4085 : C_{ps} := 4186 : C_{pw} := 380 :$

Densities

>  $rho_t := 800 : rho_s := 1200 : rho_w := 8000 :$

Flowrates

>  $F_t := 1 : F_s := 1 :$

Heat transfer coefficients

>  $U_t := 40000 : U_s := 40000 :$

Thermal conductivity of tube-wall

>  $k_w := 109 :$

Length of heat exchanger

>  $L := 1 :$

Internal and external diameter of inner tube, and internal diameter of outer tube.

>  $D_i := 0.05 : D_o := 0.06 : D_{is} := 0.1 :$

## Partial Differential Equations Derived From Energy Balances

Tube wall heat balance

$$\begin{aligned} > pde1 := \frac{\pi}{4} (D_o^2 - D_i^2) C_{pw} rho_w \left( \frac{\partial}{\partial t} T_w(x, t) \right) = U_t \pi D_i (T_t(x, t) - T_w(x, t)) \\ & - U_s \pi D_o (T_w(x, t) - T_s(x, t)) + k_w \frac{\pi}{4} \cdot (D_o^2 - D_i^2) \left( \frac{\partial^2}{\partial x^2} T_w(x, t) \right) : \end{aligned}$$

Tube-side heat balance

$$\begin{aligned} > pde2 := rho_t C_{pt} \frac{\pi}{4} D_i^2 \left( \frac{\partial}{\partial t} T_t(x, t) \right) = -C_{pt} F_t \left( \frac{\partial}{\partial x} T_t(x, t) \right) - \pi D_i U_t (T_t(x, t) \\ & - T_w(x, t)) : \end{aligned}$$

Shell-side heat balance

$$\begin{aligned} > pde3 := rho_s C_{ps} \frac{\pi}{4} (D_{is}^2 - D_o^2) \left( \frac{\partial}{\partial t} T_s(x, t) \right) = C_{ps} F_s \left( \frac{\partial}{\partial x} T_s(x, t) \right) \\ & + \pi D_o U_s (T_w(x, t) - T_s(x, t)) : \end{aligned}$$

Initial and boundary conditions

>  $ibc := T_s(x, 0) = 300, T_s(L, t) = 300,$

$T_t(x, 0) = 360, T_t(0, t) = 360,$

$T_w(x, 0) = 330, D_1(T_w)(0, t) = 0, D_1(T_w)(L, t) = 0 :$

## Solution and Results

>  $sol := pdsolve(\{pde1, pde2, pde3\}, \{ibc\}, numeric, time = t, range = 0 .. L) :$

>  $p1 := sol:-plot(T_t, x = 0.5 L, t = 0 .. 5, axes = boxed, color = black, legend = ["Tube-side liquid"]) :$

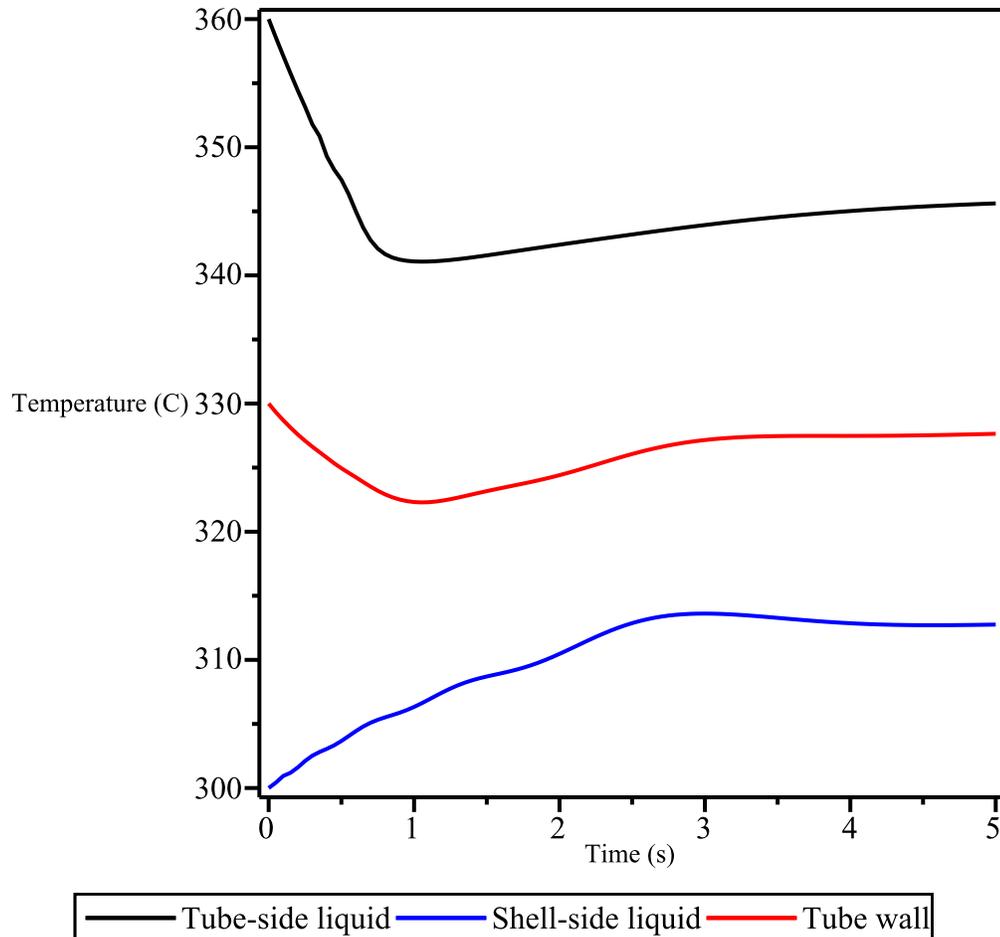
$p2 := sol:-plot(T_s, x = 0.5 L, t = 0 .. 5, axes = boxed, color = blue, legend = ["Shell-side liquid"]) :$

$p3 := sol:-plot(T_w, x = 0.5 L, t = 0 .. 5, axes = boxed, color = red, legend = ["Tube wall"]) :$

$plots[display](p1, p2, p3, labels = ["Time (s)", "Temperature (C)"], title$

$= "Temperature Halfway Along Heat Exchanger", titlefont = [Helvetica, 18, Bold])$

## Temperature Halfway Along Heat Exc



```
> p1 := sol:-plot(Tt, t = 5, x = 0 .. L, axes = boxed, color = black, legend = ["Tube-side liquid"]) :  
p2 := sol:-plot(Ts, t = 5, x = 0 .. L, axes = boxed, color = blue, legend = ["Shell-side liquid"]) :  
p3 := sol:-plot(Tw, t = 5, x = 0 .. L, axes = boxed, color = red, legend = ["Tube wall"]) :  
plots[display](p1, p2, p3, labels = ["Distance (m)", "Temperature (C)"], title  
= "Temperature Profile Along Heat Exchanger", titlefont = [Helvetica, 18, Bold])
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# Temperature Profile Along Heat Exch

