

$\mathbb{N}$  can be divided in an even and an odd number sequence:  $2n$  and  $2n-1$ .

Any even number can be divided by 2 or a multiple of 2 so that the result will be odd. So, if we can prove the conjecture of Collatz for the odd numbers then also the even numbers are proven.

Let's analyse what is happening when we apply the Collatz sequence to the odd numbers. The first thing we must do is multiply them by 3 and add 1. Next, we must divide those results by 2 because the result is always even.

In other words, we may divide the odd number sequence in two as well. Let's start with the sequence  $2n-1$ , these are all odd numbers. We can divide that sequence in  $4n-1$  and  $4n-3$ .

Now we apply the Collatz sequence on these odd number sequences:

$$3(4n - 3) + 1 = 12n - 8 \text{ we divide 2 times by 2 and we get } 3n - 2$$

$$3(4n - 1) + 1 = 12n - 2 \text{ this is a subset or subsequence of } 3n - 2 \text{ so if } 4n - 3 \text{ is proven then } 4n - 1 \text{ is proven.}$$

The table below shows the odd number sequence split each time in unique sub-sequences. This process can be continued infinitely.

$2n-1$							
$4n-1$	$4n-3$						
$8n-1$	$8n-3$	$8n-5$	$8n-7$				
$16n-1$	$16n-3$	$16n-5$	$16n-7$	$16n-9$	$16n-11$	$16n-13$	$16n-15$

After applying the first step for odd numbers according to the Collatz function we get:

$3n-1$							
$12n-2$	$12n-8$						
$24n-2$	$24n-8$	$24n-14$	$24n-20$				
$48n-2$	$48n-8$	$48n-14$	$48n-20$	$48n-26$	$48n-32$	$48n-38$	$48n-44$

After applying the step for even numbers of the Collatz function (divide by 2) that many time as possible we get:

$3n-1$							
$6n-1$	$3n-2$						
$12n-1$	$3n-1$	$12n-7$	$6n-5$				
$24n-1$	$6n-1$	$24n-7$	$12n-5$	$24n-13$	$3n-2$	$24n-19$	$12n-11$

Now we look at the sequences in the second row.  $12n-2$  is a subset  $3n-2$ , so if we can prove the Collatz conjecture for  $3n-2$  then it's proven for  $12n-2$ . When both are proven then  $2n-1$  (or  $3n-1$ ) is proven.

Now we look at the following rows. The third row are all subsets of  $3n-1$  or  $3n-1$  itself. So, we can still say that if we can prove the Collatz conjecture for  $3n-2$  then all other sequences are proven. In the fourth row happens the same, all are subsets or sub-sequences, except for  $3n-2$  that still needs to be proven.

When we take out most of the sub-sequences or subsets then we get the following table:

Odd number sequence	Produces	$2^n$	$(2^{2^n}+2)/6$	Common difference	$(4^n-1)/3$
$2n-1$	$3n-1$	2	1	1	
$4n-3$	$3n-2$	4	3	1	1
$8n-3$	$6n-2$ (subset)	8	3	5	
$16n-11$	$3n-2$	16	11	5	5
$32n-11$	$6n-2$ (subset)	32	11	21	
$64n-43$	$3n-2$	64	43	21	21
$128n-43$	$6n-2$ (subset)	128	43	85	
$256n-171$	$3n-2$	256	171	85	85
$512n-171$	$6n-2$ (subset)	512	171	341	
$1024-683$	$3n-2$	1024	683	341	341

What we see next is that there are two sequences, the even number sequence  $2^n$  that multiplies with  $n$  and the sequence  $(2^{2^n}+2)/6$  that is deducted from the even number sequence. The sequence  $2^n$  needs to be divided in 2 as well to eliminate the subsets  $(6n-2)$ . This leaves us with the sequence  $4^n$ .

The common difference between the sequence  $4^n$  and  $(2^{2^n}+2)/6$  is  $(4^n-1)/3$ .

When we put that common difference sequence in the Collatz sequence, we get the even number sequence  $4^n$ .

The result for any  $n$  in that sequence can be divided by 2 that many times as needed to reach 1 and therefore the Collatz conjecture is true.

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